

**GENERAL ATOMICS ENERGY PRODUCTS**  
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**MODELING ENERGY  
STORAGE CAPACITORS  
WHEN A CAPACITOR IS  
NOT A CAPACITOR**

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# MODELING ENERGY STORAGE CAPACITORS OR WHEN A CAPACITOR IS NOT A CAPACITOR

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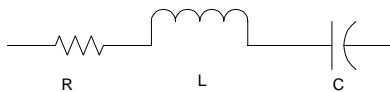
## Abstract

Energy storage capacitors are used extensively in pulsed power systems as primary or intermediate energy storage units. In very fast, low inductance systems internal construction of the capacitor can have significant affect on system performance. This is true in systems such as fast Marx generators, EMP systems and lasers. We will discuss here some of the methods that can be used to model capacitors in circuit analysis programs such as SPICE. This information can lead to better predictions of the performance of the system prior to construction and can provide insight in specifying the correct capacitor. Typically energy storage capacitors have been modeled as either just a capacitance or a series RLC circuit. This works fine for a large majority of applications, but for faster systems a more detailed model is required that reflects affects of capacitor construction. We will discuss modeling of several types of capacitors and the relationship to capacitor construction.

## I. INTRODUCTION

Modeling of pulsed power systems is critical to the prediction of performance in large pulsed power systems. Understanding the proper model can be critical for accurate prediction of performance. Development of models based on actual physical structure of the component can assist in the design of optimal devices for demanding applications.

In many cases a simple RLC model is used to represent the behavior of a capacitor. This can be a very useful tool as long as the limitations of the model are understood. The RLC model consists of a resistor, inductor and capacitor in series as shown in Figure 1.



**Figure 1.** Lumped series component circuit model often used to model pulse capacitors.

The impedance of this model is given by

$$Z = R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}$$

## II. DC LINK CAPACITOR

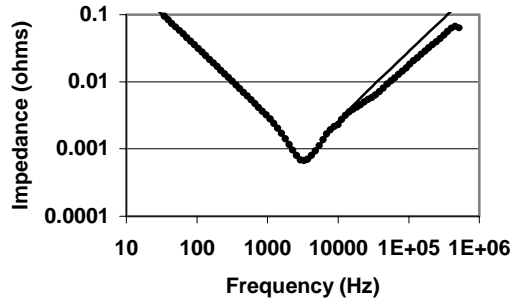
As an example of modeling a capacitor using the RLC model and it's limitations consider the model 36477 capacitor manufactured by GAEP for use in extremely high power, high power density inverter applications. This capacitor is rated at 49000  $\mu$ F, 1200-volt and stores 35 kJ. The impedance of the capacitor was measured with a bridge as a function of frequency. A low inductance fixture was used to make the measurement as shown in Figure 2.



**Figure 2.** Impedance measurement made using a low inductance fixture on the DC link capacitor.

The results of these measurements are shown in Figure 3. Also shown is a lumped RLC fit to the capacitor. Notice that the model fits very well below and through the resonance area but above 10 kHz there is an offset from the RLC model. The steel capacitor case causes this offset. At low frequencies the magnetic field produced by the current flow in the capacitor passes through the case (This case is stainless steel and is non-magnetic). Above 10 kHz, eddy currents begin to flow in the case and force the magnetic field to stay inside the case. This reduces the inductance and increases losses.

The model of this capacitor can be improved by adding a parallel RL section in series with the RLC model to represent the induced currents in the capacitor case

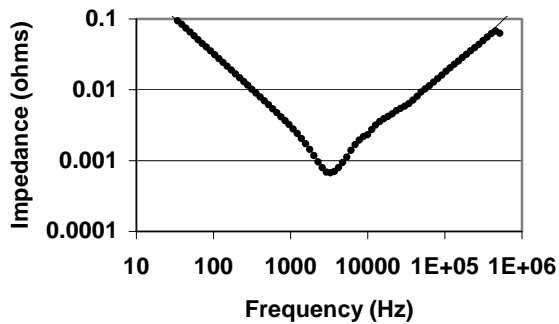


**Figure 3.** This graph shows the measured impedance magnitude and the RLC model.

Model of a Capacitor with can skin effect added

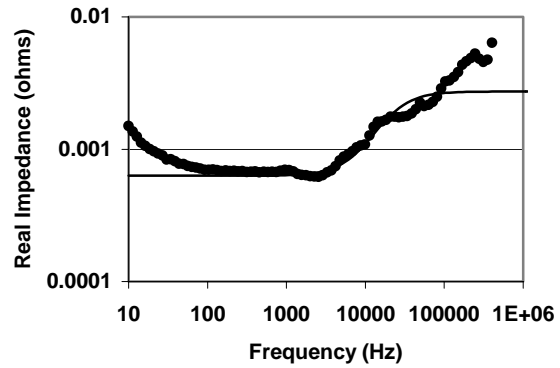
$$Z = R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} + \frac{R_p \cdot j \cdot \omega \cdot L_p}{R_p + j \cdot \omega \cdot L_p}$$

This model is shown in Figure 4 and matches the data very well over the frequency range of interest.



**Figure 4.** This graph shows the measured impedance magnitude compared to the improved model with the parallel RL section.

Figure 5 shows the real component of the impedance comparing the measured and model. At low frequencies the losses increase because of dielectric losses in the capacitor. At high frequencies the losses continue to go up probably due to skin effect, which is not included in this model. At high frequencies and pulse repetition rates, plastic case capacitors, such as those shown in Figure 5, are often used to avoid eddy current losses.



**Figure 5.** This graph shows the real component of the impedance compared to the improved model with the parallel RL section.



**Figure 6.** Plastic case capacitors avoid eddy current losses.

### III. WAVE EFFECTS

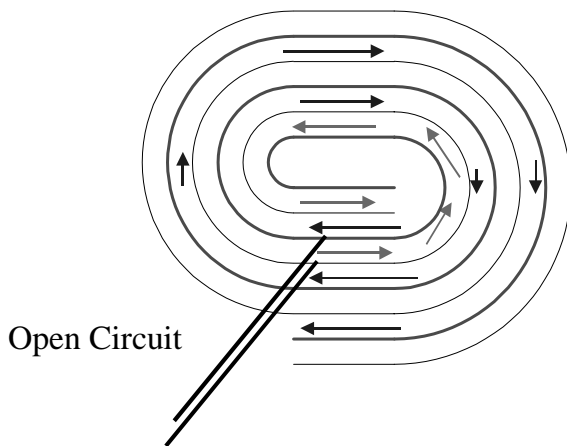
High voltage capacitors used for peaking capacitors, Marx generators and laser discharge circuits are designed to produce fast high current pulses. Internal structure of the capacitor must be considered to accurately model the performance. High voltage capacitors are manufactured by winding layers of foil/metallized film, film and paper on a winding machine like the one shown in figure 6.



**Figure 7.** Winding machine for fabrication of capacitors.

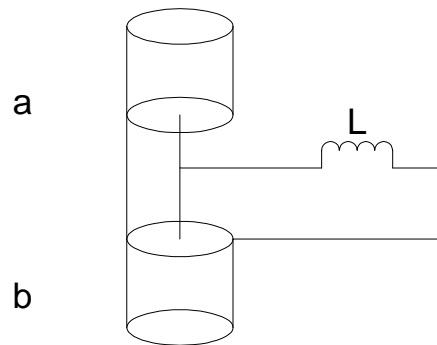
In many capacitors, such as the GAEP Type S capacitor, a tab is placed between the layers to connect to the capacitor electrode.

The winding produces a transmission line structure like that shown in figure 8. The black lines represent the tabs connecting to the capacitor electrodes. The arrows in the figure show the path of an inward-traveling wave entering the winding by the tabs. The red arrows represent the path of the wave traveling to the center of the winding. (There is also a wave that travels outward but this is not shown.) Because the winding being modeled is flattened, the center of the winding represents a relatively low inductance. The wave travels back to the outside in the second dielectric, as represented by the blue arrows.



**Figure 8** a section view of a flattened capacitor winding. The arrows indicate a wave traveling to the center of the winding and the blue arrows indicate the same wave traveling back out the “back side” of the winding.

This structure can be modeled using transmission lines as shown in figure 9. Transmission line **a** represents the line from the tab to the center of the winding and then the back side of the winding going out. Transmission line **b** represents the path of the other wave, which travels outward from the tabs to the outside end of the winding.



**Figure 9.** The capacitor winding can be modeled as a pair of transmission lines.

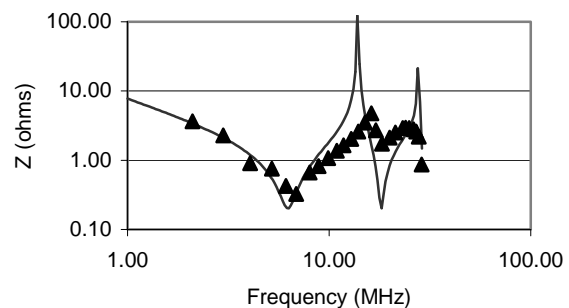
An analytical model for this can be built by using the driving point impedance of an open circuit transmission line which is:

$$Z(\omega, d) := Z \cdot \frac{\left( \begin{array}{c} -j \cdot 2 \cdot \frac{\omega}{v} \cdot d \\ 1 + e \end{array} \right)}{-j \cdot 2 \cdot \frac{\omega}{v} \cdot d - 1 - e}$$

Combining the two transmission lines with external inductance to represent the tab inductance and resistance the impedance of the capacitor becomes:

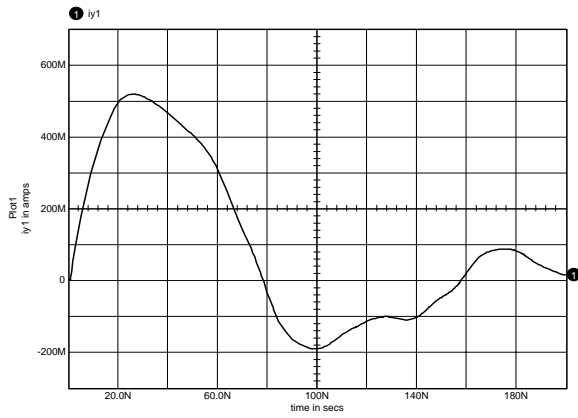
$$Z_{\text{parallel}}(\omega, a, b) := \frac{\frac{Z(\omega, a) \cdot Z(\omega, b)}{3}}{\frac{Z(\omega, a)}{3} + Z(\omega, b)} + 1j\omega \cdot L_{\text{bus}} + R$$

A plastic case capacitor was measured to verify this model. The results are shown in figure 10. Note the multiple resonances’ produced by the transmission line structure. The reduced magnitude of the resonances compared to the model is probably due to high frequency losses in the Kraft paper/polyester dielectric and skin effect in the conductors.



**Figure 10** shows the analytical model and the measured results.

Using this model in spice we calculated the waveform for a short circuit discharge of the capacitor. You can see how the transmission line structure changes the waveform from a traditional damped ringing discharge.

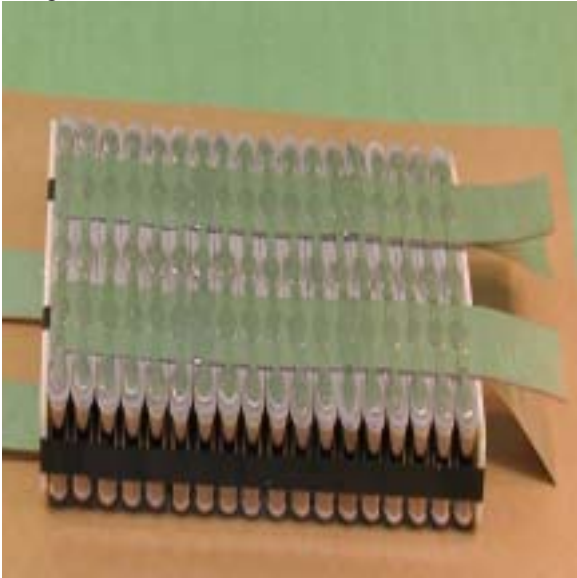


**Figure 11** shows a SPICE model calculation of a shorted capacitor ring down.

We are currently working on advancing this model for other configurations including multiple tabs.

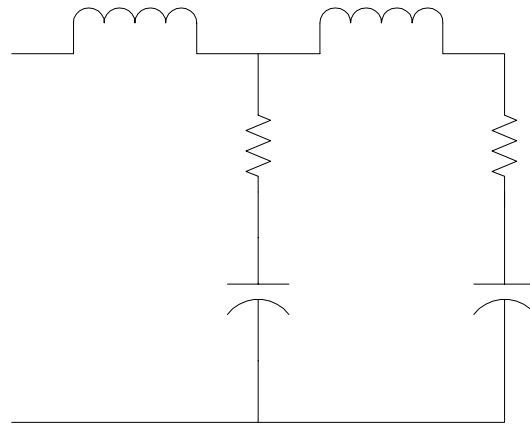
#### IV. TOPOLOGY

Typically capacitors are manufactured by connecting windings in series or parallel. Very large energy storage capacitors may have many parallel windings as is shown in figure 12.



**Figure 12** shows a metallized film capacitor assembly which consists of multiple windings.

These types of structures can be modeled as capacitors interconnected with inductors as in a pulse forming network. This structure can lead to frequency dependent losses in the capacitor. Figure 13 shows a circuit model of two windings connected in parallel. At low frequencies the inductance has little effect and the capacitor electrically looks like a capacitor with the two resistors in parallel in series. At high frequencies the inductors dominate. The inductor in series with the winding on the right tends to isolate that winding and the most of the current flow through the first winding and its internal resistance. This increases the losses.



**Figure 13** is an equivalent circuit of two windings in parallel.

Figure 14 shows the resulting impedance magnitude and the real part of the impedance of the assembly.

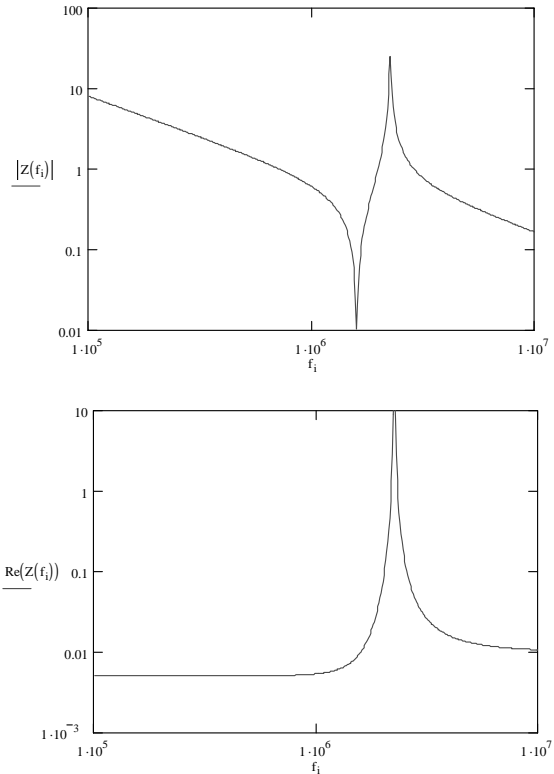


Figure 14 shows the impedance and the real component of the driving point impedance of the winding assembly shown in figure 11.

#### V. CONCLUSION

In many cases more advanced models of the capacitors should be used to accurately represent the capacitors impact on circuit performance. We have outlined some of these issues and presented ways to model different types of capacitors. Such models can also be used to help improve the design of capacitors for demanding applications.